Chapter 48

THE LE CHATELIER PRINCIPLE OF THE CAPITAL MARKET EQILIBRIUM

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Abstract

This paper purports to provide a theoretical underpinning for the problem of the Investment Company Act. The theory of the Le Chatelier Principle is well-known in thermodynamics: The system tends to adjust itself to a new equilibrium as far as possible. In capital market equilibrium, added constraints on portfolio investment on each stock can lead to inefficiency manifested in the right-shifting efficiency frontier. According to the empirical study, the potential loss can amount to millions of dollars coupled with a higher risk-free rate and greater transaction and information costs.

Keywords: Markowitz model; efficient frontiers; with constraints; without constraints; Le Chatelier Principle; thermodynamics; capital market equilibrium; diversified mutual funds; quadratic programming; investment company act

48.1. Introduction

In the wake of a growing trend of deregulation in various industries (e.g. utility, banking, and airline), it becomes more and more important to study the responsiveness of the market to the exogenous perturbations as the system is gradually constrained. According to the law of thermodynamics, the system tends to adjust itself to a new equilibrium by counteracting the change as far as possible. This law, the Le Chatelier's Principle, was applied to economics by Samuelson (1949, 1960, 1970), Silberberg (1971, 1974, 1978), and to a class of spatial equilibrium models: linear programming, fixed demand, quadratic programming, full-fledged spatial equilibrium model by Labys and Yang (1996). Recently, it has been applied to optimal taxation by Diamond and Mirrlees (2002).

According to subchapter M of the Investment Company Act of 1940, a diversified mutual fund cannot have more than 5 percent of total assets invested in any single company and the acquisition of securities does not exceed 10 percent of the acquired company's value. This diversification rule, on the one hand, reduces the portfolio risk according to the fundamental result of investment theory. On the other hand, more and more researchers begin to raise questions as to the potential inefficiency arising from the Investment Company Act (see Elton and Gruber, 1991; Roe, 1991; Francis, 1993; Kohn, 1994). With the exception of the work by Cohen and Pogue (1967), Frost and Savarino (1988), and Lovisek and Yang (1997), there is very little evidence to refute or favor this conjecture.

Empirical findings (e.g. Loviscek and Yang, 1997) suggest that over 300 growth mutual funds evaluated by Value Line shows that the average weight for the company given the greatest share of a fund's assets was 4.29 percent. However, the Le Chatelier's Principle in terms of the Investment Company Act has not been scrutinized in the literature of finance. The objective of this paper is to investigate the Le Chatelier Principle applied to the capital market equilibrium in the framework of the Markowitz portfolio selection model.

48.2. The Le Chatelier Principle of the Markowitz Model

In a portfolio of n securities, Markowitz (1952, 1956, 1959, 1990, 1991) formulated the portfolio selection model in the form of a quadratic programming as shown below

$$\min_{x_i x_j} v = \sum_{i\in I} x_i^2 \sigma_{ii} + \sum_{i\in I}\sum_{j\in J} x_i x_j \sigma_{ij} \tag{48.1}$$

$$\text{subject to} \sum_{i\in I} r_i x_i \geq k \tag{48.2}$$

$$\sum_{i\in I} x_i = 1 \tag{48.3}$$

$$x_i \geq 0 \ \forall i \in I, \tag{48.4}$$

where x_i = proportion of investment in security i
σ_{ii} = variance of rate of return of security i
σ_{ij} = covariance of rate of return of security i and j
r_i = expected rate of return of security i
k = minimum rate of return of the portfolio
I and J are sets of positive integers

The resulting Lagrange function is therefore

$$L = v + \lambda\left(k - \sum r_i x_{ij}\right) + \gamma\left(1 - \sum x_i\right) \tag{48.5}$$

The solution to the Markowitz is well-known (1959). The Lagrange multiplier of constraint from Equation (48.2) assumes the usual economic interpretation: change in total risk in response to an infinitesimally small change in k while all other decision variables adjust to their new equilibrium levels, i.e. $\lambda = \mathrm{d}v/\mathrm{d}k$. Hence, the Lagrange multiplier is of utmost importance in determining the shape of the efficiency frontier curve in the capital market. Note that values of x_{is} are unbounded between 0 and 1 in the Markowitz model. However, in reality, the proportion of investment on each security many times cannot exceed a certain percentage to ensure adequate diversification. As the maximum investment proportion on each security decreases from 99 percent to 1 percent, the solution to the portfolio selection model becomes more constrained, i.e. the values of optimum xs are bounded within a narrower range as the constraint is tightened. Such impact on the objective function v is straight forward: as the system is gradually constrained, the limited freedom of optimum xs gives rise to a higher and higher risk level as k is increased. For example, if parameter k is increased gradually, the Le Chatelier Principle implies that in the original Markowitz minimization system, isorisk contour has the smallest curvature to reflect the most efficient adjustment mechanism:

$$\text{abs}\left(\frac{\partial^2 v}{\partial k^2}\right) \leq \text{abs}\left(\frac{\partial^2 v^*}{\partial k^2}\right) \leq \text{abs}\left(\frac{\partial v^{**}}{\partial k^2}\right), \tag{48.6}$$

where v^* and v^{**} are the objective function (total portfolio risk) corresponding to the additional constrains of $x_\text{i} \leq s^*$ and $x_\text{i} \leq s^{**}$ for all i and $s^* > s^{**}$ represent different investment proportions allowed under V^* and V^{**}, and abs denotes absolute value. Via the envelope theorem (Dixit, 1990), we have

$$\begin{aligned}\mathrm{d}\{L(x_i(k),k) = v(x_i(k))\}/dk &= \partial\{L(x_i,k) \\ &= v(x_i(k))\}/\partial k \\ &= \lambda | x_i = x_i(k)\end{aligned} \tag{48.7}$$

hence Equation (48.6) can be rewritten as

$$\text{abs}\left(\frac{\partial\lambda}{\partial k}\right) \leq \text{abs}\left(\frac{\partial\lambda^*}{\partial k}\right) \leq \text{abs}\left(\frac{\partial\lambda^{**}}{\partial k}\right) \tag{48.8}$$

Equation (48.8) states that the Lagrange multiplier of the original Markowitz portfolio selection model is less sensitive to an infinitesimally small change in k than that of the model when the constraints are gradually tightened. Note that the Lagrange multiplier λ is the reciprocal of the slope of the efficiency frontier curve frequently drawn in investment textbooks. Hence, the original

Markowitz model has the steepest slope for a given set of $x_i s$. However, the efficiency frontier curve of the Markowitz minimization system has a vertical segment corresponding to a range of low ks and a constant v. Only within this range do the values of optimum xs remain equal under various degrees of constraints. Within this range constraint Equation (48.2) is not active, hence the Lagrange multiplier is 0. As a result, equality relation holds for Equation (48.8). Outside this range, the slopes of the efficiency frontier curve are different owing to the result of Equation (48.8).

48.3. Simulation Results

To verify the result implied by the Le Chatelier, we employ a five-stock portfolio with $x_i \leq 50$ percent and $x_i \leq 40$ percent. The numerical solutions are reported in Table 1. An examination of Table 1 indicates that the efficiency frontier curve is vertical and all optimum xs are identical between $0.001 \leq k \leq 0.075$. After that, the solutions of xs begin to change for the three models. Note that the maximum possible value for x_4 remains 0.4 throughout the simulation for $k > 0.075$ for the model with the tightest constraint $x_i \leq 0.4$. In the case of $x_i \leq 0.5$, a relatively loosely constrained Markowitz system, all the optimum values of decision variables remain the same as the original Markowitz model between $0.01 \leq k \leq 0.1$. Beyond that range, the maximum value of x_4 is limited to 0.5. As can be seen from Table 1, the total risk v responds less volatile to the change in k in the original unconstrained Markowitz system than

Table 48.1.

LEAST-CONSTRAINED SOLUTION (Original Markowitz Model)							SOLUTION WITH $x_i \leq 0.5$						SOLUTION WITH $x_i \leq 0.4$					
K(%)	$v(10^{-5})$	x_1%	x_2%	x_3%	x_4%	x_5%	$v(10^{-5})$	x_1%	x_2%	x_3%	x_4%	x_5%	$v(10^{-5})$	x_1%	x_2%	x_3%	x_4%	x_5%
1	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0
2	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0
3	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0
4	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0
5	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0
6	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0	257.2	39.19	0	31.87	28.94	0
7	260.8	35.02	0	32.6	32.38	0	260.8	35.02	0	32.6	32.38	0	260.8	35.02	0	32.6	32.38	0
7.5	274.8	30.54	0	32.77	36.69	0	274.8	30.54	0	32.77	36.69	0	274.8	30.54	0	32.77	36.69	0
8	299.3	25.82	0	33.27	40.91	0	299.3	25.82	0	33.27	40.91	0	300.5	24.91	0	34.55	40	5.39
8.5	333.1	21.65	0	33.26	43.63	1.45	333.1	21.65	0	33.26	43.63	1.45	340.2	20.42	0	35.34	40	4.24
9	371.2	17.82	0	32.92	45.73	3.53	371.2	17.82	0	32.92	45.73	3.53	387.7	15.93	0	36.13	40	7.94
9.5	413.2	14.05	0	32.53	47.64	5.79	413.2	14.05	0	32.53	47.64	5.79	443	11.44	0	36.92	40	11.64
10	459	9.68	0.58	32.17	49.59	7.98	459	9.68	0.58	32.17	49.59	7.98	506.2	6.95	0	37.71	40	15.34
10.5	508.3	4.83	1.96	31.44	51.56	10.2	509.5	4.25	2.1	32.23	50	11.42	576.7	1.23	1.93	37.7	40	19.15
11	560.9	0	3.53	30.46	53.55	12.46	567.5	0	2.66	32.03	50	15.31	656.5	0	0.21	36.45	40	23.34
11.5	619.9	0	1.34	27.91	55.8	14.95	637.4	0	0	30.39	50	19.62	751.7	0	0	31.79	40	28.22
12	687.5	0	0	24.31	58.11	17.58	724.5	0	0	25.39	50	24.62	866.3	0	0	26.79	40	33.22
12.5	765.4	0	0	19.02	60.68	20.3	826.7	0	0	20.52	50	29.48	995.2	0	0	21.91	40	38.09
13	854.3	0	0	13.73	63.2	23.07	949.7	0	0	15.53	50	34.48						
13.5	954	0	0	8.45	65.72	25.83	1086.8	0	0	10.65	50	39.45						
14	1064.6	0	0	3.16	68.25	28.59	1243.3	0	0	5.73	50	44.28						
14.5	1309.1	0	0	0	55.63	44.37	1417.7	0	0	0.79	50	49.21						
15	2847.3	0	0	0	20	80												
15.29	4402	0	0	0	0	100												

that in the constrained systems. In other words, the original Markowitz minimization system guarantees a smallest possible total risk due to the result of the Le Chatelier's Principle: a thermodynamic system (risk-return space) can most effectively adjust itself to the parametric change (temperature or minimum rate of return of a portfolio or k) if it is least constrained.

48.4. Policy Implications of the Le Chatelier's Principle

As shown in the previous section, the efficiency frontier curve branches off to the right first for the most binding constraint of $x_i \leq s^{**}$. Consequently, the tangency point between the efficiency frontier curve and a risk-free rate on the vertical axis must occur at a higher risk-free rate. As the value of maximum investment proportion for each stock s decreases, i.e. the constraint becomes more binding; there is a tendency for the risk-free rate to be higher in order to sustain an equilibrium (tangency) state. Second, one can assume the existence of a family of isowelfare functions (or indifference curves) in the v–k space. The direct impact of the Le Chatelier Principle on the capital market equilibrium is a lower level of welfare measure due to the right branching-off of the efficiency frontier curve. In sum, as the constraint on the maximum investment proportion is tighter, the risk-free rate will be higher and investors in the capital market will in general experience a lower welfare level. In particular, the 5 percent rule carries a substantial cost in terms of shifting of the efficiency frontier to the right. The study by Loviscek and Yang (1997) based on a 36-security portfolio indicates the loss is about 1 to 2 percentage points and the portfolio risk is 20 to 60 percent higher. Given the astronomical size of a mutual fund, 1 to 2 percentage point translates into millions of dollars potential loss in daily return. Furthermore, over diversification would incur greater transaction and information cost, which speaks against the Investment Company Rule.

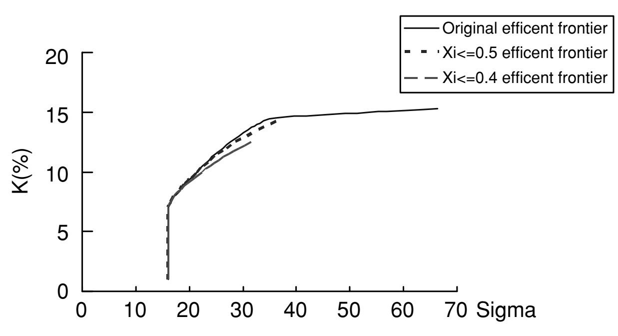

Figure 48.1. Efficent Frontier

48.5. Conclusion

In this paper, we apply the Le Chatelier Principle in thermodynamics to the Markowitz's portfolio selection model. The analogy is clear: as a thermodynamic system (or the capital market in the v–k space) undergoes some parametric changes (temperature or minimum portfolio rate of change k), the system will adjust most effectively if it is least constrained. The simulation shows that as the constraint becomes more and more tightened, the optimum investment proportions are less and less sensitive. Via the envelope theorem, it is shown that investors will be experiencing a higher risk-free rate and a lower welfare level in the capital market, if a majority of investors in the capital market experience the same constraint, i.e. maximum investment proportion on each security. Moreover, the potential loss in daily returns can easily be in millions on top of much greater transaction and information costs.

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